International Journal of Theoretical Physics, Vol. 40, No. 4, 2001

Squeezed Vector and Its Phase Distribution in a Deformed Hilbert Space

P. K. Das[1]

Received August 11, 2000

In this paper we study squeezed vectors, squeezed Kerr vectors, and their phase distributions in a deformed Hilbert space.

1. INTRODUCTION

Two-photon processes are extremely interesting in quantum optics, for the high degree of correlation between the photons in a pair may lead to the generation of nonclassical states of the electromagnetic field such as squeezed states or number states. This system offers a unique chance to study the interaction of a single mode of the electromagnetic field with a source of correlated pairs of photons under controlled conditions. Squeezed states are the eigenstates of a linear combination of annihilation and creation operators of electromagnetic field. These are pure quantum-mechanical states of light, which have reduced fluctuations in one field quadrature when compared with coherent states. These states are studied extensively (Mehta *et al.*, 1992) as they can considerably reduce noise in any signal. These states are also known as two-photon coherent states.

In generalizing two-photon processes in a deformed Hilbert space, we face a setback as the conventional Weyl–Heisenberg approach of defining squeezing operator to genereate squeezed vector fails. We adopt the idea of Solomon and Katriel (1990) to generate squeezed vector in the deformed Hilbert space.

The work is organized as follows. In Section 2, we discuss preliminaries and notations. In Section 3, we discuss generation of squeezed vectors in H_q. In Section 4, we define squeezed Kerr vectors in H_q, its coherent vector representation and quasiprobability distribution of squeezed Kerr vectors. In Section 5, after briefly describing phase vectors in H_q we study phase distribution of squeezed and squeezed Kerr vectors. In Section 6, we give a conclusion.

[1] Physics and Applied Mathematics Unit, Indian Statistical Institute, 203, B. T. Road, Calcutta-700 035, India; e-mail: daspk@isical.ac.in

0020-7748/01/0400-0807$19.50/0

2. PRELIMINARIES AND NOTATIONS

We consider the set

$$H_q = \left\{ f : f(z) = \sum a_n z^n \text{ where } \sum [n]!|a_n|^2 < \infty \right\},$$

where $[n] = (1 - q^n)/(1 - q)$, $0 < q < 1$.

For $f, g \in H_q$, $f(z) = \sum_{n=0}^{\infty} a_n z^n$, $g(z) = \sum_{n=0}^{\infty} b_n z^n$, we define addition and scalar multiplication as follows:

$$(f + g)(z) = f(z) + g(z) = \sum_{n=0}^{\infty} (a_n + b_n) z^n \tag{1}$$

and

$$(\lambda \circ f)(z) = \lambda \circ f(z) = \sum_{n=0}^{\infty} \lambda\, a_n z^n. \tag{2}$$

It is easily seen that H_q forms a vector space with respect to usual pointwise scalar multiplication and pointwise addition by (1) and (2). We observe that $e_q(z) = \sum_{n=0}^{\infty} z^n/[n]!$ belongs to H_q.

Now we define the inner product of two functions $f(z) = \sum a_n z^n$ and $g(z) = \sum b_n z^n$ belonging to H_q as

$$(f, g) = \sum [n]! \bar{a}_n b_n. \tag{3}$$

Corresponding norm is given by

$$\| f \|^2 = (f, f) = \sum [n]! |a_n|^2 < \infty.$$

With this norm, derived from the inner product, it can be shown that H_q is a complete normed space. Hence H_q forms a Hilbert space.

In a recent paper (Das, 1998, 1999a) we have proved that the set $\{z^n/\sqrt{[n]!}, n = 0, 1, 2, 3, \ldots\}$ forms a complete orthonormal set. If we consider the actions

$$T f_n = \sqrt{[n]}\, f_{n-1} \quad \text{and} \quad T^* f_n = \sqrt{[n+1]}\, f_{n+1} \tag{4}$$

on H_q, where T is the backward shift and its adjoint T^* is the forward shift operator on H_q and $f_n(z) = z^n/\sqrt{[n]!}$, then the solution (Das, 1998, 1999a) of the following eigenvalue equation:

$$T f_\alpha = \alpha f_\alpha \tag{5}$$

is given by

$$f_\alpha = e_q(|\alpha|^2)^{-1/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{[n]!}} f_n. \tag{6}$$

We call f_α a *coherent vector* in H_q.

3. GENERATION OF SQUEEZED VECTORS

Squeezed vector is generated by the action of $T - \alpha T^*$ on an arbitrary vector f_β in H_q (Solomon and Katriel, 1990) and which satisfies the following equation:

$$(T - \alpha T^*) f_\beta = 0, \tag{7}$$

where

$$f_\beta(z) = \sum_{n=0}^{\infty} a_n z^n = \sum_{n=0}^{\infty} a_n \sqrt{[n]!}\, f_n(z) \tag{8}$$

or,

$$f_\beta = \sum_{n=0}^{\infty} a_n \sqrt{[n]!}\, f_n.$$

We have

$$\begin{aligned} T f_\beta &= \sum_{n=0}^{\infty} a_n \sqrt{[n]!} T f_n \\ &= \sum_{n=1}^{\infty} a_n \sqrt{[n]!}\sqrt{[n]}\, f_{n-1} \\ &= \sum_{n=0}^{\infty} a_{n+1} \sqrt{[n+1]!}\sqrt{[n+1]}\, f_n \end{aligned} \tag{9}$$

and

$$\begin{aligned} \alpha T^* f_\beta &= \sum_{n=0}^{\infty} \alpha a_n \sqrt{[n]!} T^* f_n \\ &= \sum_{n=0}^{\infty} \alpha a_n \sqrt{[n]!}\sqrt{[n+1]}\, f_{n+1} \end{aligned} \tag{10}$$

Now from (7)–(10) we observe that a_n satisfies the following difference equation:

$$a_{n+1}\sqrt{[n+1]!}\sqrt{[n+1]} = \alpha a_{n-1}\sqrt{[n-1]!}\sqrt{[n]}. \tag{11}$$

That is,

$$a_{n+2} = \alpha \frac{\sqrt{[n]!}}{\sqrt{[n+2]!}} \frac{\sqrt{[n+1]}}{\sqrt{[n+2]}} a_n \tag{12}$$

and

$$a_1 = 0. \tag{13}$$

Hence,

$$a_2 = \alpha \frac{\sqrt{[0]!}}{\sqrt{[2]!}} \frac{\sqrt{[1]}}{\sqrt{[2]}} a_0$$

$$a_4 = \alpha \frac{\sqrt{[2]!}}{\sqrt{[4]!}} \frac{\sqrt{[3]}}{\sqrt{[4]}} a_2 = \alpha^2 \frac{\sqrt{[2]!}\sqrt{[0]!}}{\sqrt{[4]!}\sqrt{[2]!}} \frac{\sqrt{[3]}\sqrt{[1]}}{\sqrt{[4]}\sqrt{[2]}} a_0$$

$$a_6 = \alpha \frac{\sqrt{[4]!}}{\sqrt{[6]!}} \frac{\sqrt{[5]}}{\sqrt{[6]}} a_4 = \alpha^3 \frac{\sqrt{[4]!}\sqrt{[2]!}\sqrt{[0]!}}{\sqrt{[6]!}\sqrt{[4]!}\sqrt{[2]!}} \frac{\sqrt{[5]}\sqrt{[3]}\sqrt{[1]}}{\sqrt{[6]}\sqrt{[4]}\sqrt{[2]}} a_0.$$

and so on. Thus,

$$a_{2n} = \alpha^n \frac{1}{\sqrt{[2n]!}} \frac{[2n-1]!!}{\sqrt{[2n]!!}} a_0$$

and

$$a_1 = a_3 = a_5 = \cdots = a_{2n-1} = 0.$$

Thus, f_β satisfying (7) has the form

$$f_\beta = \sum_{n=0}^{\infty} a_n \sqrt{[n]!}\, f_n = a_0 \sum_{n=0}^{\infty} \alpha^n \sqrt{\frac{[2n-1]!!}{[2n]!!}}\, f_{2n}. \tag{14}$$

To normalize, we have

$$1 = (f_\beta,\ f_\beta) = |a_0|^2 \sum_{n=0}^{\infty} |\alpha|^{2n} \frac{[2n-1]!!}{[2n]!!}. \tag{15}$$

Thus, aside from a trivial phase we have

$$a_0 = \left[\sum_{n=0}^{\infty} |\alpha|^{2n} \frac{[2n-1]!!}{[2n]!!} \right]^{-1/2} \tag{16}$$

and the squeezed vector f_β takes the form

$$f_\beta = \left[\sum_{n=0}^{\infty} |\alpha|^{2n} \frac{[2n-1]!!}{[2n]!!} \right]^{-1/2} \sum_{n=0}^{\infty} \alpha^n \sqrt{\frac{[2n-1]!!}{[2n]!!}}\, f_{2n}. \tag{17}$$

4. SQUEEZED KERR VECTORS

Squeezed Kerr vector ϕ_β^{K} in H_q is defined by

$$\phi_\beta^{\mathrm{K}} = e_q^{\frac{i}{2}\gamma N(N-1)} f_\beta, \tag{18}$$

where f_β in H_q is a squeezed vector given by (17), γ is the Kerr constant and $N = T^*T$, T is the backwardshift (4).

Now,

$$\begin{aligned}
\phi_\beta^{\mathrm{K}} &= e_q^{\frac{i}{2}\gamma N(N-1)} f_\beta \\
&= e_q^{\frac{i}{2}\gamma N(N-1)} \left[\sum_{n=0}^{\infty} |\alpha|^{2n} \frac{[2n-1]!!}{[2n]!!}\right]^{-1/2} \sum_{n=0}^{\infty} \alpha^n \sqrt{\frac{[2n-1]!!}{[2n]!!}} f_{2n} \\
&= \left[\sum_{n=0}^{\infty} |\alpha|^{2n} \frac{[2n-1]!!}{[2n]!!}\right]^{-1/2} \sum_{n=0}^{\infty} \alpha^n \sqrt{\frac{[2n-1]!!}{[2n]!!}} e_q^{\frac{i}{2}\gamma [2n]([2n]-1)} f_{2n} \\
&= \sum_{n=0}^{\infty} \left\{ \left[\sum_{n=0}^{\infty} |\alpha|^{2n} \frac{[2n-1]!!}{[2n]!!}\right]^{-1/2} \alpha^n \sqrt{\frac{[2n-1]!!}{[2n]!!}} e_q^{\frac{i}{2}\gamma [2n]([2n]-1)} \right\} f_{2n}.
\end{aligned} \tag{19}$$

4.1. Coherent Vector Representation

To obtain the coherent vector representation of squeezed Kerr vector ϕ_β^{K} we try to calculate the matrix element $(f_{\alpha'}, \phi_\beta^{\mathrm{K}})$ that contains all important information about the vector ϕ_β^{K}.

The matrix element is obtained by the following elegant method (Král, 1990). We utilize the completeness relation (Das, 1998, 1999a) of coherent vectors in H_q

$$I = \frac{1}{2\pi} \int_{\alpha \in \mathbb{C}} d\mu(\alpha) \,|\, f_\alpha >< f_\alpha |, \tag{20}$$

where

$$d\mu(\alpha) = e_q(|\alpha|^2) e_q(-|\alpha|^2)\, d_q|\alpha|^2 \; d\theta \tag{21}$$

where $\alpha = re^{i\theta}$, and obtain

$$\begin{aligned}
\left(f_{\alpha'}, \phi_\beta^{\mathrm{K}}\right) &= (f_{\alpha'}, U f_\beta) \\
&= \frac{1}{2\pi} \int_{\alpha_1 \in \mathbb{C}} d\mu(\alpha_1)\, (f_{\alpha'}, U |\, f_{\alpha_1} >< f_{\alpha_1} |\, f_\beta) \\
&= \frac{1}{2\pi} \int_{\alpha_1 \in \mathbb{C}} d\mu(\alpha_1)\, (f_{\alpha_1}, \, f_\beta)(f_{\alpha'}, U f_{\alpha_1})
\end{aligned} \tag{22}$$

where $U \equiv e_q^{\frac{i}{2}\gamma N(N-1)}$.

Now,

$$(f_{\alpha_1}, \, f_\beta) = \sum_{m=0,n=0}^{\infty} e_q(|\alpha_1|^2)^{-1/2} \left[\sum_{n=0}^{\infty} |\alpha|^{2n} \frac{[2n-1]!!}{[2n]!!}\right]^{-1/2}$$

$$\times \frac{\bar{\alpha}_1^m}{\sqrt{[m]!}} \alpha^n \sqrt{\frac{[2n-1]!!}{[2n]!!}} (f_m, f_{2n})$$

$$= \sum_{n=0}^{\infty} e_q(|\alpha_1|^2)^{-1/2} \left[\sum_{n=0}^{\infty} |\alpha|^{2n} \frac{[2n-1]!!}{[2n]!!} \right]^{-1/2}$$

$$\times \frac{1}{\sqrt{[2n]!}} \sqrt{\frac{[2n-1]!!}{[2n]!!}} \bar{\alpha}_1^{2n} \alpha^n \tag{23}$$

and

$$(f_{\alpha'}, U f_{\alpha_1}) = e_q(|\alpha_1|^2)^{-1/2} e_q(|\alpha'|^2)^{-1/2} \sum_{n=0}^{\infty} e_q^{\frac{i}{2}\gamma[2n]([2n]-1)} \frac{(\bar{\alpha}'\alpha_1)^n}{[n]!}. \tag{24}$$

Hence, we have

$$(f_{\alpha_1}, f_\beta)(f_{\alpha'}, U f_{\alpha_1}) = \sum_{m=0,n=0}^{\infty} e_q(|\alpha_1|^2)^{-1} e_q(|\alpha'|^2)^{-1/2}$$

$$\times \left[\sum_{n=0}^{\infty} |\alpha|^{2n} \frac{[2n-1]!!}{[2n]!!} \right]^{-1/2} \frac{1}{\sqrt{[2m]!}} \sqrt{\frac{[2m-1]!!}{[2m]!!}}$$

$$\times \bar{\alpha}_1^{2m} \alpha^m e_q^{\frac{i}{2}\gamma[2n]([2n]-1)} \frac{(\bar{\alpha}'\alpha_1)^n}{[n]!}. \tag{25}$$

Thus,

$$\left(f_{\alpha'}, \phi_\beta^{\mathrm{K}}\right) = \frac{1}{2\pi} \int_{\alpha_1 \in \mathbb{C}} d\mu(\alpha_1)\, (f_{\alpha_1}, f_\beta)(f_{\alpha'}, U f_{\alpha_1})$$

$$= \sum_{m=0,n=0}^{\infty} e_q(|\alpha'|^2)^{-1/2} \left[\sum_{n=0}^{\infty} |\alpha|^{2n} \frac{[2n-1]!!}{[2n]!!} \right]^{-1/2} \frac{1}{\sqrt{[2m]!}}$$

$$\times \sqrt{\frac{[2m-1]!!}{[2m]!!}} \alpha^m\, (\bar{\alpha}')^n e_q^{\frac{i}{2}\gamma[2n]([2n]-1)} \frac{1}{[n]!} \frac{1}{2\pi}$$

$$\times \int_{\alpha_1 \in \mathbb{C}} d\mu(\alpha_1)\, e_q(|\alpha_1|^2)^{-1} \bar{\alpha}_1^{2m} \alpha_1^n$$

$$= \sum_{m=0,n=0}^{\infty} e_q(|\alpha'|^2)^{-1/2} \left[\sum_{n=0}^{\infty} |\alpha|^{2n} \frac{[2n-1]!!}{[2n]!!} \right]^{-1/2} \frac{1}{\sqrt{[2m]!}}$$

$$\times \sqrt{\frac{[2m-1]!!}{[2m]!!}}\alpha^m(\bar{\alpha}')^n e_q^{\frac{i}{2}\gamma[2n]([2n]-1)} \frac{1}{[n]!}\frac{1}{2\pi}\int_0^\infty d_q r^2\, e_q(-r^2) r^{2m+n}$$
$$\times \int_0^{2\pi} d\theta\, e^{i(n-2m)}$$
$$= \sum_{n=0}^{\infty} e_q(|\alpha'|^2)^{-1/2}\left[\sum_{n=0}^{\infty}|\alpha|^{2n}\frac{[2n-1]!!}{[2n]!!}\right]^{-1/2}\frac{1}{\sqrt{[2n]!}}\sqrt{\frac{[2n-1]!!}{[2n]!!}}\alpha^n$$
$$\times (\bar{\alpha}')^{2n} e_q^{\frac{i}{2}\gamma[4n]([4n]-1)}\frac{1}{[2n]!}\int_0^\infty d_q r^2\, e_q(-r^2)\,(r^2)^{2n}$$
$$= \sum_{n=0}^{\infty} e_q(|\alpha'|^2)^{-1/2}\left[\sum_{n=0}^{\infty}|\alpha|^{2n}\frac{[2n-1]!!}{[2n]!!}\right]^{-1/2}\frac{1}{\sqrt{[2n]!}}\sqrt{\frac{[2n-1]!!}{[2n]!!}}\alpha^n$$
$$\times (\bar{\alpha}')^{2n} e_q^{\frac{i}{2}\gamma[4n]([4n]-1)}\frac{1}{[2n]!}\int_0^\infty d_q x\, e_q(-x)(x)^{2n}$$
$$= \sum_{n=0}^{\infty} e_q(|\alpha'|^2)^{-1/2}\left[\sum_{n=0}^{\infty}|\alpha|^{2n}\frac{[2n-1]!!}{[2n]!!}\right]^{-1/2}\frac{1}{\sqrt{[2n]!}}\sqrt{\frac{[2n-1]!!}{[2n]!!}}\alpha^n$$
$$\times (\bar{\alpha}')^{2n} e_q^{\frac{i}{2}\gamma[4n]([4n]-1)}, \tag{26}$$

where we have taken $x = r^2$ and utilized the fact $\int_0^\infty d_q x\, e_q^{-x} x^n = [n]!$ (Gray and Nelson, 1990).

4.2. Quasiprobability Distribution

The *quasiprobability distribution*, known as the Q function, for the squeezed Kerr vector is introduced in the following form:

$$\begin{aligned} Q(\alpha_1) &= \frac{1}{\pi}\left(f_{\alpha_1}, \left|\phi_\beta^{\mathrm{K}} >< \phi_\beta^{\mathrm{K}}\right| f_{\alpha_1}\right) \\ &= \frac{1}{\pi}\left(f_{\alpha_1}, (\phi_\beta^{\mathrm{K}},\, f_{\alpha_1})\phi_\beta^{\mathrm{K}}\right) \\ &= \frac{1}{\pi}\left|\left(f_{\alpha_1}, \phi_\beta^{\mathrm{K}}\right)\right|^2 \qquad (27) \\ &= \frac{1}{\pi}\left|\sum_{n=0}^{\infty} e_q(|\alpha_1|^2)^{-1/2}\left[\sum_{n=0}^{\infty}|\alpha|^{2n}\frac{[2n-1]!!}{[2n]!!}\right]^{-1/2}\frac{1}{\sqrt{[2n]!}}\sqrt{\frac{[2n-1]!!}{[2n]!!}}\alpha^n\right. \\ &\quad \left.\times (\bar{\alpha}_1)^{2n} e_q^{\frac{i}{2}\gamma[4n]([4n]-1)}\right|^2. \end{aligned}$$

5. PHASE DISTRIBUTION

In this section, we describe the phase distribution of squeezed and squeezed Kerr vectors. To do this we introduce first the phase vectors and its distributions in details.

5.1. Phase Vectors

To obtain the phase vector we consider first the Susskind–Glogower type *phase operator* $P = (q^n + T^*T)^{-1/2}T$ and try to find the solution of the following eigenvalue equation:

$$P f_\beta = \beta f_\beta, \tag{28}$$

where

$$f_\beta(z) = \sum_{n=0}^{\infty} a_n z^n = \sum_{n=0}^{\infty} a_n \sqrt{[n]!}\, f_n(z). \tag{29}$$

We arrive at

$$\begin{aligned} f_\beta &= \sum_{n=0}^{\infty} a_n \sqrt{[n]!}\, f_n \\ &= a_0 \sum_{n=0}^{\infty} \beta^n \sqrt{\frac{(q+[0])(q^2+[1])(q^3+[2])\cdots(q^n+[n-1])}{[n]!}}\, f_n, \end{aligned}$$

where $\beta = |\beta| e^{i\theta}$ is a complex number. For details we refer to (Das, 1999b).

These vectors are normalizable in a strict sense only for $|\beta| < 1$.

Now, if we take $a_0 = 1$ and $|\beta| = 1$, we have

$$f_\beta = \sum_{n=0}^{\infty} e^{in\theta} \sqrt{\frac{(q+[0])(q^2+[1])(q^3+[2])\cdots(q^n+[n-1])}{[n]!}}\, f_n. \tag{30}$$

Henceforth, we shall denote this vector as

$$f_\theta = \sum_{n=0}^{\infty} e^{in\theta} \sqrt{\frac{(q+[0])(q^2+[1])(q^3+[2])\cdots(q^n+[n-1])}{[n]!}}\, f_n, \tag{31}$$

$0 \le \theta \le 2\pi$ and call f_θ a *phase vector* in H_q.

The phase vectors f_θ are neither normalizable nor orthogonal, but form a complete set and yield the following resolution of the identity:

$$I = \frac{1}{2\pi} \int_X \int_0^{2\pi} d\nu(x,\theta)\, |\, f_\theta >< f_\theta|, \tag{32}$$

where

$$d\nu(x,\theta) = d\mu(x)\, d\theta. \tag{33}$$

For a proof of completeness of phase vectors we refer to Das (2000).

We use the vectors f_θ to associate, to a given density operator ρ, a phase distribution as follows:

$$\begin{aligned} P(\theta) &= \frac{1}{2\pi}(f_\theta, \rho f_\theta) \\ &= \frac{1}{2\pi}\sum_{m,n=0}^{\infty}\sqrt{\frac{(q+[0])\cdots(q^m+[m-1])}{[m]!}} \\ &\quad \times \sqrt{\frac{(q+[0])\cdots(q^n+[n-1])}{[n]!}} e^{i(n-m)}(f_m, \rho f_n). \end{aligned} \tag{34}$$

Where $P(\theta)$ is positive, owing to the positivity of ρ, and is normalized

$$\int_X \int_0^{2\pi} P(\theta)\, d\nu(x,\theta) = 1, \tag{35}$$

where

$$d\nu(x,\theta) = d\mu(x)\, d\theta \tag{36}$$

for,

$$\begin{aligned} \int_X \int_0^{2\pi} P(\theta)\, d\nu(x,\theta) &= \int_X d\mu(x) \sum_{m,n=0}^{\infty}\sqrt{\frac{(q+[0])\cdots(q^m+[m-1])}{[m]!}} \\ &\quad \times \sqrt{\frac{(q+[0])\cdots(q^n+[n-1])}{[n]!}}\frac{1}{2\pi}\int_0^{2\pi} \\ &\quad \times e^{i(m-n)\theta}\, d\theta(f_m, \rho f_n) \\ &= \int_X d\mu(x) \sum_{n=0}^{\infty}\frac{(q+[0])\cdots(q^n+[n-1])}{[n]!}(f_n, \rho f_n) \\ &= \sum_{n=0}^{\infty}(f_n, \rho f_n) \\ &= 1. \end{aligned} \tag{37}$$

In particular, the *phase distribution* over the window $0 \le \theta \le 2\pi$ for any vector

f is then defined by

$$
\begin{aligned}
P(\theta) &= \frac{1}{2\pi}(f_\theta, |f>< f| f_\theta) \\
&= \frac{1}{2\pi}|(f_\theta,\ f)|^2.
\end{aligned}
\tag{38}
$$

5.2. Phase Distribution of Squeezed Vector

To obtain the *phase distribution* over the window $0 \le \theta \le 2\pi$ for the squeezed vector f_β in (17) we take the density operator $\rho = |f_\beta >< f_\beta|$ and calculate $P(\theta)$ as follows:

$$
\begin{aligned}
P(\theta) &= \frac{1}{2\pi}(f_\theta, |f_\beta >< f_\beta| f_\theta) \\
&= \frac{1}{2\pi}|(f_\theta,\ f_\beta)|^2.
\end{aligned}
\tag{39}
$$

The *phase representation* $(f_\theta,\ f_\beta)$ of the squeezed vector f_β is calculated as follows:

$$
\begin{aligned}
(f_\theta,\ f_\beta) &= \left[\sum_{n=0}^{\infty} |\alpha|^{2n} \frac{[2n-1]!!}{[2n]!!}\right]^{-1/2} \sum_{n=0}^{\infty} e^{-2in\theta} \\
&\quad \times \sqrt{\frac{(q+[0])(q^2+[1])(q^3+[2])\cdots(q^{2n}+[2n-1])}{[2n]!}} \\
&\quad \times \alpha^n \sqrt{\frac{[2n-1]!!}{[2n]!!}}.
\end{aligned}
\tag{40}
$$

From (39) and (40) we have the phase distribution $P(\theta)$ of the squeezed vector f_β as

$$
\begin{aligned}
P(\theta) &= \frac{1}{2\pi}|(f_\theta,\ f_\beta)|^2 \\
&= \frac{1}{2\pi}\left|\left[\sum_{n=0}^{\infty} |\alpha|^{2n} \frac{[2n-1]!!}{[2n]!!}\right]^{-1/2} \sum_{n=0}^{\infty} e^{-2in\theta}\right. \\
&\quad \left. \times \sqrt{\frac{(q+[0])(q^2+[1])(q^3+[2])\cdots(q^{2n}+[2n-1])}{[2n]!}}\alpha^n \sqrt{\frac{[2n-1]!!}{[2n]!!}}\right|^2.
\end{aligned}
\tag{41}
$$

5.3. Phase Distribution of Squeezed Kerr Vector

To obtain the *phase distribution* of squeezed Kerr vector ϕ_β^{K} over the window $0 \leq \theta \leq 2\pi$ we calculate the *phase representation* $(f_\theta, \phi_\beta^{\mathrm{K}})$ as follows:

$$\begin{aligned}\left(f_\theta, \phi_\beta^{\mathrm{K}}\right) = {} & \left[\sum_{n=0}^{\infty} |\alpha|^{2n} \frac{[2n-1]!!}{[2n]!!}\right]^{-1/2} \sum_{n=0}^{\infty} e^{-2in\theta} \\ & \times \sqrt{\frac{(q+[0])(q^2+[1])(q^3+[2])\cdots(q^{2n}+[2n-1])}{[2n]!}} \\ & \times \alpha^n \sqrt{\frac{[2n-1]!!}{[2n]!!}} e_q^{\frac{i}{2}\gamma[2n]([2n]-1)}.\end{aligned} \tag{42}$$

From (42) we have the phase distribution $P(\theta)$ of the squeezed Kerr vector ϕ_β^{K} as

$$\begin{aligned}P(\theta) &= \frac{1}{2\pi}\left|\left(f_\theta, \phi_\beta^{\mathrm{K}}\right)\right|^2 \\ &= \frac{1}{2\pi}\Bigg|\left[\sum_{n=0}^{\infty} |\alpha|^{2n} \frac{[2n-1]!!}{[2n]!!}\right]^{-1/2} \sum_{n=0}^{\infty} e^{-2in\theta} \\ &\quad \times \sqrt{\frac{(q+[0])(q^2+[1])(q^3+[2])\cdots(q^{2n}+[2n-1])}{[2n]!}} \\ &\quad \times \alpha^n \sqrt{\frac{[2n-1]!!}{[2n]!!}} e_q^{\frac{i}{2}\gamma[2n]([2n]-1)}\Bigg|^2.\end{aligned} \tag{43}$$

6. CONCLUSION

In conclusion, we have thus generalized the notion of squeezed vector in a deformed Hilbert space and described its phase distribution. This notion is then utilized to define squeezed Kerr vector, its coherent vector reprensentation, quasiprobability distribution, and its corresponding phase distribution in the deformed Hilbert space.

REFERENCES

Das, P. K. (1998). Eigenvectors of Backwardshift on a Deformed Hilbert Space, *International Journal of Theoretical Physics* **37**(9), 2363.

Das, P. K. (1999a). Erratum: Eigenvectors of Backwardshift on a Deformed Hilbert Space. *International Journal of Theoretical Physics* **38**(7), 2063.

Das, P. K. (1999b). Phase distribution of Kerr vectors in a deformed Hilbert space, *International Journal of Theoretical Physics* **38**(6), 1807.

Das, P. K. (2000). Erratum: Phase distribution of Kerr vectors in a deformed Hilbert space. *International Journal of Theoretical Physics* **39** (4).

Gray, R. W. and Nelson, C. A. (1990). A completeness relation for the q-analogue coherent states by q-integration. *Journal of Physics A: Mathematical and General* **23**, L945–L950.

Král, P. (1990). Kerr interaction with displaced and squeezed Fock states. *Physical Review A* **42**, 4177–4192.

Mehta, C. L., Roy, A. K., and Saxena, G. M. (1992). Eigenstates of two-photon annihilation operators. *Physical Review A* **46**, 1565–1572.

Solomon, A. I. and Katriel, J. (1990). On q-squeezed states. *Journal of Physics A: Mathematical and General* **23**, 11209–11212.